1. A nursery has a sack containing a large number of coloured beads of which 14% are coloured red.

Aliya takes a random sample of 18 beads from the sack to make a bracelet.

(a) State a suitable binomial distribution to model the number of red beads in Aliya's bracelet.

(1)

(b) Use this binomial distribution to find the probability that

(i) Aliya has just 1 red bead in her bracelet,

(ii) there are at least 4 red beads in Aliya's bracelet.

(3)

(c) Comment on the suitability of a binomial distribution to model this situation.

(1)

After several children have used beads from the sack, the nursery teacher decides to test whether or not the proportion of red beads in the sack has changed.
She takes a random sample of 75 beads and finds 4 red beads.

(d) Stating your hypotheses clearly, use a 5% significance level to carry out a suitable test for the teacher.

(4)

(e) Find the *p*-value in this case.

(1)

a) let R be the number of red beads in Aliya's bracelet

$R \sim B(18, 0.14)$ 1

b) i) $P(R=1) = 0.19403... = 0.194$ (3sf) 1

ii) $P(R \geq 4) = 1 - P(R \leq 3) = 1 - 0.76184...$ 1 $= 0.2381...$
$= 0.238$ (3sf) 1

c) a condition for a binomial distribution is that p is constant. so there must be a large number of beads in the bag such that removing 18 doesnt significantly change p. 1

two tailed ∴ halve significance level

d) $H_0: p = 0.14$ $H_1: p \neq 0.14$ 1

Assume Ho correct
let X = # of red beads in the sample

sig. level
$= 2.5\% = 0.025$

$X \sim B(75, 0.14)$ 1

1
$P(X \leq 4) = 0.01506... < 0.025$

- Sufficient evidence to reject Ho 1
- There is evidence to suggest that the proportion of red beads was changed. 1

two tailed so don't say increased/decreased.

e) p-value is $2 \times 0.01506... = 0.0301...$
$= 0.030$ (2sf) 1

2. Past information shows that 25% of adults in a large population have a particular allergy.

Rylan believes that the proportion that has the allergy differs from 25%

He takes a random sample of 50 adults from the population. two tailed

Rylan carries out a test of the null hypothesis H_0: $p = 0.25$ using a 5% level of significance.

(a) Write down the alternative hypothesis for Rylan's test.

(1)

(b) Find the critical region for this test.
You should state the probability associated with each tail, which should be as close to 2.5% as possible.

(4)

(c) State the actual probability of incorrectly rejecting H_0 for this test.

(1)

Rylan finds that 10 of the adults in his sample have the allergy.

(d) State the conclusion of Rylan's hypothesis test.

(1)

a) $H_1: p \neq 0.25$ (1)

b) let X be the number of adults with the allergy.
Assume H_0 correct: $X \sim B(50, 0.25)$ (1)

significance level for each tail $= 2.5\% = 0.025$

$P(X \leq 5) = 0.00705$
$P(X \leq 6) = 0.01939$ ← closest to 0.025
$P(X \leq 7) = 0.04526$ (1)

$P(X \geq 18) = 1 - P(X \leq 17) = 0.05512$
$P(X \geq 19) = 1 - P(X \leq 18) = 0.02873$ ← closest to 0.025
$P(X \geq 20) = 1 - P(X \leq 19) = 0.01392$ (1)

Critical region: $X \leq 6$ and $X \geq 19$ (1)

c) $0.0194 + 0.0287 = 0.048$ (2sf) (1)

d) 10 is not in the critical region ⇒ insufficient evidence to reject H_0

There is insufficient evidence to suggest that the proportion of adults with the allergy is different from 25% 1

link to context!

3. George throws a ball at a target 15 times.
Each time George throws the ball, the probability of the ball hitting the target is 0.48

The random variable X represents the number of times George hits the target in 15 throws.

(a) Find

(i) $P(X = 3)$

(ii) $P(X \geqslant 5)$

(3)

George now throws the ball at the target 250 times.

(b) Use a normal approximation to calculate the probability that he will hit the target more than 110 times.

(3)

a) $X \sim B(15, 0.48)$ ①

(i) $P(X=3) = 0.019668...$
$= 0.0197$ (3sf) ①

(ii) $P(X \geq 5) = 1 - P(X \leq 4)$
$= 0.92013...$
$= 0.920$ (3sf) ①

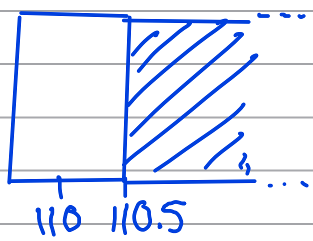
b) let Y be the number of times George hits the target.

$\mu = np = 250 \times 0.48 = 120$
$\sigma = \sqrt{np(1-p)} = \sqrt{62.4}$

$Y \sim N(120, \sqrt{62.4}^2)$ ①

$P(X > 110)$

$\approx P(Y > 110.5)$ ①



$= 0.88544...$
$= 0.885$ (3sf) ①

4. A manufacturer uses a machine to make metal rods.

The length of a metal rod, L cm, is normally distributed with

- a mean of 8 cm
- a standard deviation of x cm

Given that the proportion of metal rods less than 7.902 cm in length is 2.5%

(a) show that $x = 0.05$ to 2 decimal places.

(2)

(b) Calculate the proportion of metal rods that are between 7.94 cm and 8.09 cm in length.

(1)

The **cost** of producing a single metal rod is 20p

A metal rod

- where $L < 7.94$ is **sold** for scrap for 5p
- where $7.94 \leqslant L \leqslant 8.09$ is **sold** for 50p
- where $L > 8.09$ is shortened for an extra **cost** of 10p and then **sold** for 50p

(c) Calculate the expected profit per 500 of the metal rods.
Give your answer to the nearest pound.

(5)

The same manufacturer makes metal hinges in large batches.

The hinges each have a probability of 0.015 of having a fault.

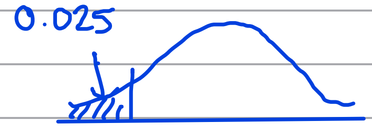
A random sample of 200 hinges is taken from each batch and the batch is accepted if fewer than 6 hinges are faulty.

The manufacturer's aim is for 95% of batches to be accepted.

(d) Explain whether the manufacturer is likely to achieve its aim.

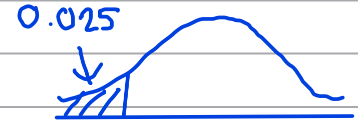
(4)

$L \sim N(8, x^2)$



$P(L < 7.902) = 0.025$

$P\left(Z < \dfrac{7.902 - 8}{x}\right) = 0.025$



$\therefore \dfrac{7.902 - 8}{x} = -1.96$ ①

$\Phi^{-1}(0.025) = -1.96$

$$\frac{7.902-8}{x}=-1.96$$

$$x=\frac{7.902-8}{-1.96}=0.05\ (2dp)\ (1)$$

b) $P(7.94<L<8.09)=P(L<8.09)-P(L<7.94)$
$=0.8490\ldots$
$=0.849\ (3dp)$ (1)

c) cost of producing 500 rods $=500\times 0.2=£100$

Number sold for scrap:
$P(L<7.94)=0.115\ (3sf)$ (1)
$0.115\times 500=57.5$ rods sold for scrap

Each sold for 5p: $57.5\times 0.05=£2.88$

Number sold for normal price:
$P(7.94<L<8.09)=0.849$
$0.849\times 500=424.5$ rods sold for normal price

Each sold for 50p: $424.5\times 0.5=£212.25$

Number shortened:
$P(L>8.09)=0.0359$ (1)
$0.0359\times 500=17.97$ rods shortened (1)

Each make profit of $50p-10p=40p$: $17.97\times 0.4=£7.19$

Total profit $=£2.88+£212.25+£7.19-£100$ (1)
$=£122.32$ (1)

d) let X be the number of hinges that are faulty

$X \sim B(200, 0.015)$ 1

$P(X<6) = P(X \leq 5)$ 1 $= 0.9176\ldots$ 1 < 0.95

So it is expected that 91.8% of batches will be accepted. Therefore the manufacturer is unlikely to achieve their aim as $91.8 < 95$ 1

5. A machine fills packets with sweets and $\frac{1}{7}$ of the packets also contain a prize.
The packets of sweets are placed in boxes before being delivered to shops.
There are 40 packets of sweets in each box.

The random variable T represents the number of packets of sweets that contain a prize in each box.

(a) State a condition needed for T to be modelled by $\mathrm{B}(40, \frac{1}{7})$

(1)

A box is selected at random.

(b) Using $T \sim \mathrm{B}(40, \frac{1}{7})$ find

(i) the probability that the box has exactly 6 packets containing a prize,

(ii) the probability that the box has fewer than 3 packets containing a prize.

(2)

Kamil's sweet shop buys 5 boxes of these sweets.

(c) Find the probability that exactly 2 of these 5 boxes have fewer than 3 packets containing a prize.

(2)

Kamil claims that the proportion of packets containing a prize is less than $\frac{1}{7}$

A random sample of 110 packets is taken and 9 packets contain a prize.

(d) Use a suitable test to assess Kamil's claim.
You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

a) The probability of a packet containing a prize is constant. 1

b) $T \sim B\left(40, \frac{1}{7}\right)$

(i) $P(T=6) = 0.1727\ldots = 0.173$ (3 s.f.) 1

(ii) $P(T<3) = P(T \leq 2)$

$= 0.061587\ldots = 0.0616$ (3 s.f.) 1

(c) Let r.v. K = number of boxes with fewer than 3 packets containing a prize.

$K \sim B(5, 0.0615...)$ 1

$\therefore P(K=2) = 0.031344... \approx 0.0313$ (3 s.f.) 1

d) Let r.v. X = number of packets containing a prize.

$X \sim B = (110, \frac{1}{7})$ 1

$H_0 : p = \frac{1}{7}$, $H_1 = p < \frac{1}{7}$ 1

$P(X \leq 9) = 0.038292...$ (which is < 0.05) 1

$\therefore$ reject H_0 since there is evidence to support Kamil's claim. 1